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Modeling of Full-Heusler Alloys within Tight-Binding Approximation: Case Study of Fe₂MnAl

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Abstract. Heusler alloys have been known for about a century, and predictions of magnetic moment values using Slater-Pauling rule have been successful for many such materials. However, such a simple counting rule has been found not to always work for all Heusler alloys. For instance, Fe₂CuAl has been found to have magnetic moment of 3.30 μ_B per formula unit although the Slater-Pauling rule suggests the value of 2 μ_B. On the other hand, a recent experiment shows that a non-stoichiometric Heusler compound Fe₂Mn₀.₅Cu₀.₅Al possesses magnetic moment of ~4 μ_B, closer to the Slater-Pauling prediction for the stoichiometric compound. Such discrepancies signify that the theory to predict the magnetic moment of Heusler alloys in general is still far from being complete. Motivated by this issue, we propose to do a theoretical study on a full-Heusler alloy Fe₂MnAl to understand the formation of magnetic moment microscopically. We model the system by constructing a density-functional-theory-based tight-binding Hamiltonian and incorporating Hubbard repulsive as well as spin-spin interactions for the electrons occupying the d-orbitals. Then, we solve the model using Green's function approach, and treat the interaction terms within the mean-field approximation. At this stage, we aim to formulate the computational algorithm for the overall calculation process. Our final goal is to compute the total magnetic moment per unit cell of this system and compare it with the experimental data.

INTRODUCTION

Heusler alloys, discovered by Fritz Heulser in 1903, are among the most exciting materials because of their magnetic properties despite that they are not made from magnetic elements. For instance, an alloy with the composition of Cu₂MnAl behaves as ferromagnet although none of its constituent elements is magnetic by itself [1, 2]. Generally, full-Heusler alloys with general formula X₂YZ (X and Y are transition metals, and Z is a main group element) are crystallized in the L2₁ structure which is composed of four interpenetrating face-centered-cubic (fcc) lattices (Fig. 1). Their atomic positions are described by Wyckoff coordinates as: X (0,0,0), Y (1/4,1/4,1/4), X (1/2,1/2,1/2) and Z (3/4,3/4,3/4) [3, 4]. Graf *et al.* [5] explains that the properties of Heusler compounds are strongly dependent on the atomic order, so a careful analysis is needed to understand the crystal structure-to-property relation of Heusler compounds.

Surprisingly, the magnetic moments of many Heusler compounds can be predicted using Slater-Pauling rule by counting the number of valance electrons, so that for full-Heusler compounds with four atoms per unit cell the magnetic moment follows the formula $m_{X_2YZ} = N_v - 24$ [5]. Recently, Gasi *et al.* [6] found that Fe₂CuAl does not follow the Slater-Pauling rule, but the non-stoichiometric Fe₂Mn₀.₅Cu₀.₅Al does [7]. This fact indicates that the theory to predict the magnetic moment of Heusler alloys may still be far from being complete. Motivated by this, we propose to do a theoretical study on a full-Heulser alloy Fe₂MnAl to understand the formation of magnetic moment microscopically using a density-functional-theory-based tight-binding model.

In this study, we model the Fe₂MnAl system by constructing a tight-binding Hamiltonian, whose parameters are obtained through density functional theory (DFT) calculation, incorporating Hubbard repulsive and spin-spin interactions for the electrons occupying the d-orbitals. The model is then solved using Green's function approach, with the interaction terms treated within mean-field approximation. Our ultimate goal of study is to compute the magnetic moment per unit cell of this system, compare it with the experimental data, and analyze the result particularly in its relation to the Slater-Pauling rule. As the introductory part of this work, however, in this paper we only present how we formulate the computational algorithm for the overall calculation process.
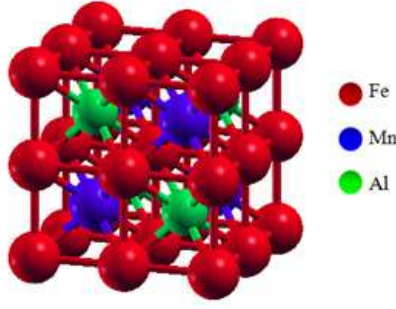
FIGURE 1. Full Heusler alloy Fe₂MnAl crystal structure.

TABLE 1. Basis states for constructing the tight-binding Hamiltonian of full-Heusler Fe₂MnAl.

$\lvert Fe_A - s \rangle$	$\lvert Fe_A - d_{xy} \rangle$	$\lvert Fe_A - d_{xz} \rangle$	$\lvert Fe_A - d_{yz} \rangle$
$\lvert Fe_A - d_{z^2} \rangle$	$\lvert Fe_A - d_{x^2-y^2} \rangle$	$\lvert Fe_B - s \rangle$	$\lvert Fe_B - d_{xy} \rangle$
$\lvert Fe_B - d_{xz} \rangle$	$\lvert Fe_B - d_{yz} \rangle$	$\lvert Fe_B - d_{z^2} \rangle$	$\lvert Fe_B - d_{x^2-y^2} \rangle$
$\lvert Fe_C - s \rangle$	$\lvert Fe_C - d_{xy} \rangle$	$\lvert Fe_C - d_{xz} \rangle$	$\lvert Fe_C - d_{yz} \rangle$
$\lvert Fe_C - d_{z^2} \rangle$	$\lvert Fe_C - d_{x^2-y^2} \rangle$	$\lvert Fe_D - s \rangle$	$\lvert Fe_D - d_{xy} \rangle$
$\lvert Fe_D - d_{xz} \rangle$	$\lvert Fe_D - d_{yz} \rangle$	$\lvert Fe_D - d_{z^2} \rangle$	$\lvert Fe_D - d_{x^2-y^2} \rangle$
$\lvert Fe_E - s \rangle$	$\lvert Fe_E - d_{xy} \rangle$	$\lvert Fe_E - d_{xz} \rangle$	$\lvert Fe_E - d_{yz} \rangle$
$\lvert Fe_E - d_{z^2} \rangle$	$\lvert Fe_E - d_{x^2-y^2} \rangle$	$\lvert Fe_F - s \rangle$	$\lvert Fe_F - d_{xy} \rangle$
$\lvert Fe_F - d_{xz} \rangle$	$\lvert Fe_F - d_{yz} \rangle$	$\lvert Fe_F - d_{z^2} \rangle$	$\lvert Fe_F - d_{x^2-y^2} \rangle$
$\lvert Fe_G - s \rangle$	$\lvert Fe_G - d_{xy} \rangle$	$\lvert Fe_G - d_{xz} \rangle$	$\lvert Fe_G - d_{yz} \rangle$
$\lvert Fe_G - d_{z^2} \rangle$	$\lvert Fe_G - d_{x^2-y^2} \rangle$	$\lvert Fe_H - s \rangle$	$\lvert Fe_H - d_{xy} \rangle$
$\lvert Fe_H - d_{xz} \rangle$	$\lvert Fe_H - d_{yz} \rangle$	$\lvert Fe_H - d_{z^2} \rangle$	$\lvert Fe_H - d_{x^2-y^2} \rangle$
$\lvert Mn_A - s \rangle$	$\lvert Mn_A - d_{xy} \rangle$	$\lvert Mn_A - d_{xz} \rangle$	$\lvert Mn_A - d_{yz} \rangle$
$\lvert Mn_A - d_{z^2} \rangle$	$\lvert Mn_A - d_{x^2-y^2} \rangle$	$\lvert Mn_B - s \rangle$	$\lvert Mn_B - d_{xy} \rangle$
$\lvert Mn_B - d_{xz} \rangle$	$\lvert Mn_B - d_{yz} \rangle$	$\lvert Mn_B - d_{z^2} \rangle$	$\lvert Mn_B - d_{x^2-y^2} \rangle$
$\lvert Mn_C - s \rangle$	$\lvert Mn_C - d_{xy} \rangle$	$\lvert Mn_C - d_{xz} \rangle$	$\lvert Mn_C - d_{yz} \rangle$
$\lvert Mn_C - d_{z^2} \rangle$	$\lvert Mn_C - d_{x^2-y^2} \rangle$	$\lvert Mn_D - s \rangle$	$\lvert Mn_D - d_{xy} \rangle$
$\lvert Mn_D - d_{xz} \rangle$	$\lvert Mn_D - d_{yz} \rangle$	$\lvert Mn_D - d_{z^2} \rangle$	$\lvert Mn_D - d_{x^2-y^2} \rangle$
$\lvert Al_A - s \rangle$	$\lvert Al_A - p_x \rangle$	$\lvert Al_A - p_y \rangle$	$\lvert Al_A - p_z \rangle$
$\lvert Al_B - s \rangle$	$\lvert Al_B - p_x \rangle$	$\lvert Al_B - p_y \rangle$	$\lvert Al_B - p_z \rangle$
$\lvert Al_C - s \rangle$	$\lvert Al_C - p_x \rangle$	$\lvert Al_C - p_y \rangle$	$\lvert Al_C - p_z \rangle$
$\lvert Al_D - s \rangle$	$\lvert Al_D - p_x \rangle$	$\lvert Al_D - p_y \rangle$	$\lvert Al_D - p_z \rangle$

MODEL

The crystal structure of the full-Heusler alloy Fe₂MnAl can be described using Wyckoff coordinates as: Fe (0,0,0), Mn (1/4,1/4,1/4), Fe (1/2,1/2,1/2) and Al (3/4,3/4,3/4). From the 88 valence orbitals involved in the unit cell of Fe₂MnAl listed in Table 1, we define the basis states to construct our tight-binding Hamiltonian that serves as the non-interacting part of our full Hamiltonian.

The full Hamiltonian itself reads as

$$H = \sum_{\vec{k}} \hat{c}_{\vec{k}}^{\dagger} \left[H_0(\vec{k}) \right] \hat{c}_{\vec{k}} - J_H \sum_{\zeta,\gamma,\alpha,\beta} \vec{s}_{\gamma,\alpha}^{(\zeta)} \cdot \vec{s}_{\gamma,\beta}^{(\zeta)} + U \sum_{\zeta,\gamma,\alpha} n_{\gamma,\alpha\uparrow}^{(\zeta)} n_{\gamma,\alpha\downarrow}^{(\zeta)}. \tag{1}$$

The first term of Equation 1 denotes the non-interacting or the kinetic part of the Hamiltonian. Here, $\left[H_0(\vec{k}) \right]$ is an 88×88 matrix defined in momentum space whose elements are constructed within tight-binding approximation using the above 88 basis states. The elements of the matrix $\left[H_0(\vec{k}) \right]$ in Equation 1 can be written as

$$H_{0\ i,j}(\vec{k}) = \epsilon_i \delta_{i,j} - t_{i,j} \sum_{\vec{\delta}_{nn}} e^{-i\vec{k}\cdot\vec{\delta}_{nn}}, \tag{2}$$

where ϵ_i denotes the on-site energy corresponding to the orbital i, $\delta_{i,j}$ a Kronecker delta, $t_{i,j}$ denotes the hopping parameter connecting orbitals i and j between nearest-neighbor atoms, and $\vec{\delta}_{nn}$ represents the nearest neighbor

translation vector. Meanwhile, $\hat{c}_{\vec{k}}^{\dagger}$ ($\hat{c}_{\vec{k}}$) is a row (column) vector whose each of its element contains the creation (annihilation) operators corresponding to each basis orbital.

In the second term of Equation 1, $\sum_{\zeta,\gamma,\alpha,\beta} \vec{s}_{\gamma,\alpha}^{(\zeta)} \cdot \vec{s}_{\gamma,\beta}^{(\zeta)}$ represents the interaction between spins of electrons occupying different d-orbitals in the same atom. ζ is index of the unit cell, while γ denotes site having d-orbitals in a unit cell. α and β denote components of d-orbitals. The dot product of two spin operators, $\vec{s}_\alpha \cdot \vec{s}_\beta$, can be expanded as

$$\vec{s}_\alpha \cdot \vec{s}_\beta = s_{x\alpha}s_{x\beta} + s_{y\alpha}s_{y\beta} + s_{z\alpha}s_{z\beta}. \tag{3}$$

n terms of the creation and annihilation operators and the Pauli matrices, the above spin dot product expands further as

$$\vec{s}_\alpha \cdot \vec{s}_\beta = \left(\frac{\hbar}{2}\right)^2 \begin{pmatrix} a_{\alpha\uparrow}^\dagger & a_{\alpha\downarrow}^\dagger \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_{\alpha\uparrow} \\ a_{\alpha\downarrow} \end{pmatrix} \begin{pmatrix} a_{\beta\uparrow}^\dagger & a_{\beta\downarrow}^\dagger \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_{\beta\uparrow} \\ a_{\beta\downarrow} \end{pmatrix} +$$
$$\left(\frac{\hbar}{2}\right)^2 \begin{pmatrix} a_{\alpha\uparrow}^\dagger & a_{\alpha\downarrow}^\dagger \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a_{\alpha\uparrow} \\ a_{\alpha\downarrow} \end{pmatrix} \begin{pmatrix} a_{\beta\uparrow}^\dagger & a_{\beta\downarrow}^\dagger \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a_{\beta\uparrow} \\ a_{\beta\downarrow} \end{pmatrix} +$$
$$\left(\frac{\hbar}{2}\right)^2 \begin{pmatrix} a_{\alpha\uparrow}^\dagger & a_{\alpha\downarrow}^\dagger \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a_{\alpha\uparrow} \\ a_{\alpha\downarrow} \end{pmatrix} \begin{pmatrix} a_{\beta\uparrow}^\dagger & a_{\beta\downarrow}^\dagger \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a_{\beta\uparrow} \\ a_{\beta\downarrow} \end{pmatrix}. \tag{4}$$

To simplify our further calculation, we choose the z-direction to be the direction of the net magnetic moment of the unit cell. Then, upon applying mean-filed approximation, the terms corresponding to the average values of the spin components along the x- and y-directions vanish. Therefore, the first and second term of Equation 2 vanish accordingly, leaving only the terms corresponding to the z components

$$\vec{s}_\alpha \cdot \vec{s}_\beta \cong \left(\frac{\hbar}{2}\right)^2 \big[\left(n_{\alpha\uparrow}\langle n_{\beta\uparrow}\rangle + n_{\beta\uparrow}\langle n_{\alpha\uparrow}\rangle - \langle n_{\alpha\uparrow}\rangle\langle n_{\beta\uparrow}\rangle\right) -$$
$$\left(n_{\alpha\uparrow}\langle n_{\beta\downarrow}\rangle + n_{\beta\downarrow}\langle n_{\alpha\uparrow}\rangle - \langle n_{\alpha\uparrow}\rangle\langle n_{\beta\downarrow}\rangle\right) -$$
$$\left(n_{\alpha\downarrow}\langle n_{\beta\uparrow}\rangle + n_{\beta\uparrow}\langle n_{\alpha\downarrow}\rangle - \langle n_{\alpha\downarrow}\rangle\langle n_{\beta\uparrow}\rangle\right) +$$
$$\left(n_{\alpha\downarrow}\langle n_{\beta\downarrow}\rangle + n_{\beta\downarrow}\langle n_{\alpha\downarrow}\rangle - \langle n_{\alpha\downarrow}\rangle\langle n_{\beta\downarrow}\rangle\right)\big], \tag{5}$$

where $a_\uparrow^\dagger a_\uparrow = n_\uparrow$ ($a_\downarrow^\dagger a_\downarrow = n_\downarrow$) denotes occupation number operator of electron with spin up (down).

The last term of Equation 1 represents Hubbard local repulsive interaction. Similar to our treatment to the second term of Equation 1, we also apply mean-field approximation to this term such that

$$n_{\gamma,\alpha\uparrow}n_{\gamma,\alpha\downarrow} \approx n_{\gamma,\alpha\uparrow}\langle n_{\gamma,\alpha\downarrow}\rangle + n_{\gamma,\alpha\downarrow}\langle n_{\gamma,\alpha\uparrow}\rangle - \langle n_{\gamma,\alpha\uparrow}\rangle\langle n_{\gamma,\alpha\downarrow}\rangle. \tag{6}$$

FORMULATION OF THE METHOD

In the kinetic part of Equation1, the matrix $\left[H_0(\vec{k})\right]$ is constructed out of 88 basis orbital states in the Fe_2MnAl unit cell, giving it a structure of

$$\left[H_0(\vec{k})\right] = \begin{bmatrix} H_{1,1} & \cdots & H_{1,88} \\ \vdots & \ddots & \vdots \\ H_{88,1} & \cdots & H_{88,88} \end{bmatrix} \tag{7}$$

The parameter values of ϵ_i and $t_{i,j}$ defining each element of $\left[H_0(\vec{k})\right]$ through Equation 2 can in principle be guessed purely intuitively with some physical considerations. However, in order to obtain more realistic values we extract those parameter values by first performing a DFT calculation, then extract the tight-binding parameters from the DFT result using a computational package called Wannier90 [8].

To solve our model, we use a standard many-body approach that relies on Green's functions. For our particular purpose, we work with retarded Green's function defined in matrix form through the Dyson equation

$$\left[G^R(\vec{k},\omega)\right] = \left[(\omega + i0^+)[I] - \left[H_0(\vec{k})\right] - \left[\Sigma(\vec{k},\omega)\right]\right]^{-1}. \tag{8}$$

Here the mean-field treatment implies that self-energy matrix becomes independent of both $\vec{k}$ and ω, that is,

$$\left[\Sigma(\vec{k},\omega)\right] \approx [\Sigma]. \tag{9}$$

$[\Sigma]$ depends on the average occupation numbers of electrons of the d-orbitals as the manifestation of the mean-field treatment of the second and third terms of the Hamiltonian defined in Equation 1. Thus, the average occupation numbers of electrons of the d-orbitals, and hence, also the self-energy matrix, are to be calculated self-consistently, starting with some initial guesses. For this purpose the parameters J_H and U are to be determined a priori.

During the iteration process we need to calculation both the partial density of states (PDOS) and the total density of states (DOS) of the system, respectively, as

$$\text{PDOS}(\omega) = -\frac{1}{\pi}\frac{1}{N}\sum_{\vec{k}} \text{Im } G_{\alpha\alpha}^R(\vec{k},\omega), \tag{10}$$

$$\text{DOS}(\omega) = -\frac{1}{\pi}\frac{1}{N}\sum_{\vec{k}}\text{Im Tr}\left[G^R(\vec{k},\omega)\right]. \tag{11}$$

N is the total number of $\vec{k}$-points in the Brillouin zone, which is equal to the number of unit cells in the system. Further, we need to determine the chemical potential, μ, of the system by imposing the appropriate total number of valence electrons, n_{total}, as a constraint, that is

$$n_{total} = \int_{-\infty}^{\infty} d\omega\,\text{DOS}(\omega)f(\omega,T,\mu), \tag{12}$$

with $f(\omega,T,\mu)$ denoting the Fermi-Dirac distribution function defined as

$$f(\omega,T,\mu) = \frac{1}{e^{\frac{\omega-\mu}{k_BT}}+1}. \tag{13}$$

The obtained chemical potential is then used to re-compute the spin-resolved average occupation numbers of the d-electrons

$$\langle n_{\alpha\uparrow(\downarrow)}\rangle = \int_{-\infty}^{\infty} d\omega\,\text{PDOS}_{\alpha\uparrow(\downarrow)}(\omega)f(\omega,T,\mu), \tag{14}$$

which are then used to re-compute the self-energy matrix $[\Sigma]$. When the difference between the initial guess and the calculated value of $[\Sigma]$ is still bigger than some tolerance, some mixture of the initial guess and the calculated value of $[\Sigma]$ is used as a new initial guess, and then the process repeats back from Equation 8 to Equation 14. The cycle stops when the convergence is achieved.

After the self-consistency is achieved, we can calculate the net up and down electron spins of the system using equation

$$\langle n_{\uparrow(\downarrow)}\rangle = \int_{-\infty}^{\infty} d\omega\,\text{DOS}_{\uparrow(\downarrow)}(\omega)f(\omega,T,\mu). \tag{15}$$

And finally, we calculate the magnetic moment of the system by the following equation

$$m = \langle n_\uparrow\rangle - \langle n_\downarrow\rangle. \tag{16}$$

For Fe_2MnAl structure, if we use the primitive unit cell, the resulting value of m already directly gives us the net magnetic moment value per formula unit of Fe_2MnAl in the unit of Bohr magneton, μ_B. The entire calculation process can be summarized in the following flow chart (Fig. 2).
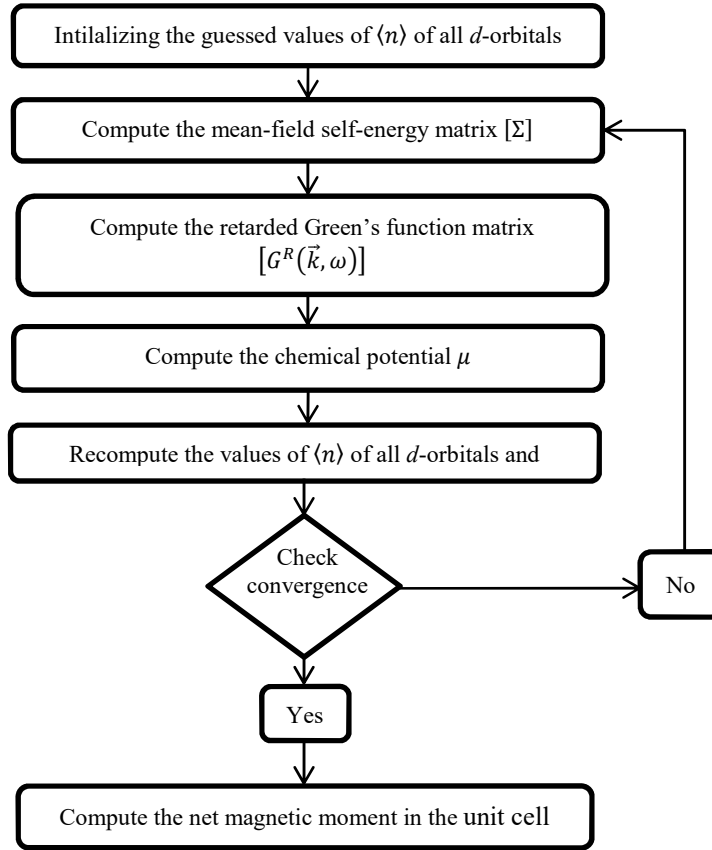

FIGURE 2. Mean-field algorithm.

SUMMARY

We have formulated the tight-binding based algorithm to calculate the net magnetic moment of Fe_2MnAl to be solved within mean-field approximation. Our next step is to develop the code and do the computation. The desired final results are to be compared with the existing experimental and other theoretical studies.

ACKNOWLEDGMENTS

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